

# A neurodynamic framework for local community extraction in networks

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To understand the structure and organization of a large-scale social, biological or technological network, it can be helpful to describe and extract local communities or modules of the network. In this article, we develop a neurodynamic framework to describe the local communities which correspond to the stable states of a neuro-system built based on the network. The quantitative criteria to describe the neurodynamic system can cover a large range of objective functions. The resolution limit of these functions enable us to propose a generic criterion to explore multi-resolution local communities. We explain the advantages of this framework and illustrate them by testing on a number of model and real-world networks.

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## I. INTRODUCTION

In recent years, networks have emerged as an invaluable tool to understanding systems of interacting objects in diverse fields including sociology, biology and technology [1, 2]. Due to the large-scale, complex nature of many systems under study, one crucial step in network analysis is the description and detection of mesoscopic structure known as modules or communities: groups of nodes that are more tightly connected to each other than they are to the rest of network that have more internal links than external (see ref.[3] for a recent review). The network communities form a distinct intermediate level and provide insight into the structure of the overall network.

Traditionally, the community detection problem is formulated as finding a “best” partition and multiple methods and heuristics have been proposed for this issue [3]. The complexity of networks makes it complicated to measure the goodness of a community structure and a variety of measures have been proposed which have been shown to produce reasonable community structure for a series of examples [4–7]. However, the partitioning methods which force every node into a community can distort the real structure of the network, in which, some nodes may only loosely connected to any community. Moreover, the popular measure modularity [4, 8] has been shown to fail to find the most natural community structure due to the resolution limit issues [9] which leads to several variants and extensions [5, 6, 10].

In a large network, a community only focus on the “local” links within it and links connecting it to a limited number of nodes of the rest of the network. The principle of determining such a local community at a time is different, but beneficial complement (view) to the partitioning methods. There has been no much work in the

literature focusing on the local community detection. Researchers have proposed local community methods aiming to look for the community around a given node which relies on the predefined knowledge [11, 12]. In a very recent study, Zhao *et al.* [13] proposed a local community extraction method by maximizing two quantitative measures via tabu search technique. The resolution limit of the proposed criteria and inefficiency of the local optimization technique partly inspire us to explore further this issue.

In this article, we propose a neurodynamic framework to describe the local organization of the links and nodes in networks that represent the local dense subunits of a system (Figure 1). The basic idea is that local communities can be captured by making it correspond to the stable states of a dynamic system. If one then starts the dynamical system in random state that are sufficiently close to one of the stable states, it should drift into one of these stable states and stay there. We therefore identify the local communities that compose the core part of a network by finding all stable states of a neurodynamic system. Moreover, we can start the system in a large number of random states, and then the frequency of each stable state can shed light on the robustness of the corresponding local community.

## II. LOCAL COMMUNITY EXTRACTION PROBLEM

Let  $G(V, E)$  denote an undirected network of  $n$  nodes; it can be represented by a symmetric  $n \times n$  adjacency matrix  $A = [A_{ij}]$ , where  $A_{ij} > 0$  if there is an edge between nodes  $i$  and  $j$  and  $A_{ij} = 0$  otherwise. If the edges have weights, the positive  $A_{ij}$ 's are the weights; if not, they are set to 1. The kernel idea of local community extraction problem is to look for a set  $S$  of nodes with a large number of links within itself and a small number of links to the rest of the network. This problem can be

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described to optimize a quantitative function, and note that the links within the complement  $S^c$  of this set do not affect the value of this function. Here we employ two quantitative functions  $W_S$  and  $Q_S$  to illustrate our framework and more other criteria including the function for describing weak definition [16], the minimum cut [17], the modularity degree [6], community density [18] on community can also be analyzed in the same manner (see Supplementary file).

Here we introduced the  $Q_S$  which is defined in sprit of the modularity  $Q$  [4, 8] and the  $W_S$  adopted by Zhao *et al.* [13] as criteria for a local community. Specifically,

$$Q_S = \frac{O_S}{O_V} - \left( \frac{O_S + B_S}{O_V} \right)^2, \quad (1)$$

and

$$W_S = |S||S^c| \left[ \frac{O_S}{|S|^2} - \frac{B_S}{|S||S^c|} \right], \quad (2)$$

where  $O_S = \sum_{i,j \in S} A_{ij}$ ,  $B_S = \sum_{i \in S, j \in S^c} A_{ij}$ . The term  $O_S$  is twice the weight of the edges (links) within  $S$ , and  $B_S$  represents connections between  $S$  and the rest of the network. The first term of  $Q_S$  is the fraction of links inside community  $S$  and the second term, in contrast, represents the expected fraction of links in the community if links were made at random but respecting node degrees in the network. The first term of  $W_S$  (in regards of the scalar  $|S||S^c|$ ) is close to the density of the community  $S$ , and the second term is expected connections between the community and the rest of the network. Intuitively, a local community should have high  $Q_S$  and/or  $W_S$ . Thus the task for resolving local communities can be changed into find such ‘good’ subnets by searching through the possible candidates for ones with relatively high  $Q_S$  and/or  $W_S$ .

In the following, we will show that the maximization of these two criteria for resolving local community can be formulated into integer quadratic (fractional) programming problems. Let’s set

$$x_i = \begin{cases} 1, & i \in S \\ 0, & i \in S^c \end{cases},$$

then  $|S| = \sum_{i=1}^n x_i$ ,  $|S^c| = n - \sum_{i=1}^n x_i$ ,  $O_S = \sum_{i=1}^n \sum_{j=1}^n A_{ij} x_i x_j$  and  $B_S = \sum_{i=1}^n \sum_{j=1}^n A_{ij} x_i (1 - x_j)$ . Thus the maximization of the objective function  $Q_S$  and  $W_S$  can be reformulated as the following optimization problems which both subject to  $x \in \{0, 1\}^n$ :

$$\begin{aligned} \min \quad & -f_Q(x) = -\frac{1}{2m} \sum_i \sum_j \left( A_{ij} - \frac{d_i d_j}{2m} \right) x_i x_j, \\ & = -\left( \frac{1}{m} \right) \frac{1}{2} x^T M_Q x. \end{aligned} \quad (3)$$

where  $M_Q = \left( A - \frac{d_i d_j}{2m} \right)$ , and

$$\begin{aligned} \min \quad & -f_W(x) = -\frac{\sum_{i=1}^n \sum_{j=1}^n A_{ij} x_i x_j (n - \sum_{k=1}^n x_k) - \sum_{i=1}^n \sum_{j=1}^n A_{ij} x_i (1 - x_j)}{\sum_{i=1}^n x_i} \\ & = -(n) \frac{x^T M_W x}{e^T x} \equiv \frac{b(x)}{a(x)} \equiv \lambda(x), \end{aligned} \quad (4)$$

where  $M_W = A - \frac{de^T}{n}$ ,  $d = (d_1, d_2, \dots, d_n)^T$ ,  $d_i = \sum_j A_{ij}$ ,  $e = (1, \dots, 1)^T$ . Since  $x^T M_W x = x^T M_W^T x$  for any  $M_W$ , we can replace the  $M_W$  with  $\frac{1}{2}(M_W + M_W^T) = A - \frac{de^T + ed^T}{2n}$ . Then the new  $M_W$  is a symmetric matrix which can ensure the convergence of the neurodynamic system introduced later.

Generally, it is a challenging task to solve the quadratic fractional programming (QFP) problem (Eq.4) directly. Fortunately, according to Dinkelbach’s theory [22], with  $e^T x > 0$ , the fractional programming problem (Eq.4) can be transformed into a parametric programming model. This model will generate a sequence of integer quadratic programs and the solutions of these programs can converge to the solution of the fractional program. Specifically, the problem Eq.(4) can be transformed into the following parametric quadratic programming problem:

$$\min \quad g(x) = (2n) \left[ -\frac{1}{2} x^T M_W x - \frac{\lambda}{2} e^T x \right] \equiv b(x) - \lambda a(x), \quad (5)$$

where  $b(x) = -x^T M_W x$  and  $a(x) = e^T x$ . By solving the quadratic program with a given  $\lambda$  and updating the  $\lambda' = \frac{b(x^*)}{a(x^*)}$  with optimal solution  $x^*$  of Eq.5 alternatively, we can finally obtain the solution of Eq.(4) (see Supplementary file for Dinkelbach theory).

We can see that the kernel problem for optimizing  $Q_S$  and  $W_S$  is to solve the unconstrained Binary Quadratic Programming (BQP) problem [20]. The BQP problem is an NP-hard problem and has a large number of important applications in a broad range of scientific fields. Various solution techniques including both exact and heuristics have been proposed such as branch and bound method, tabu search, simulated annealing and genetic algorithm [21]. However, due to the computational complexity of the problem, most of these technique is limited for large-scale problems.

### III. A NEURODYNAMIC FRAMEWORK

We further mimic this problem with a neurodynamic system — Hopfield net model. As we analyzed that an ‘optimal’ solution for the above problem is a local stable dense region. Ideally, such a solution can be related to a

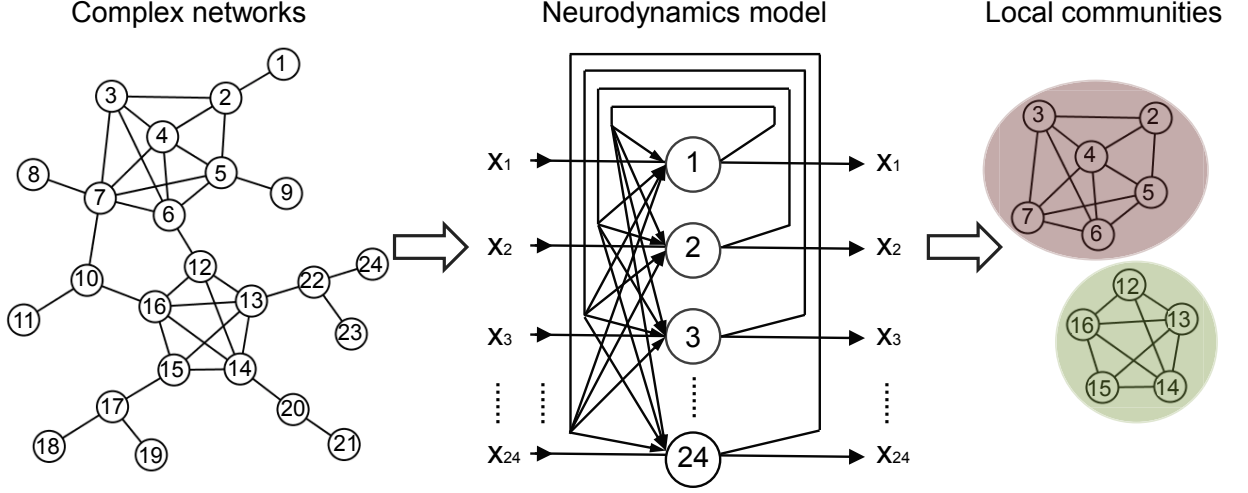


FIG. 1: Illustration of the neurodynamic framework for modeling local communities. (A) We want to describe the subnet of the network such that tight substructures have distinct properties. (B) The key to model the local communities is to build a proper neurodynamic system  $N(M, T)$  based on the structure of network  $G(V, E)$ , of which, the stable states correspond to the local tight subunits. (C) By running the system from a random initial state, the system can converge to a stable state which corresponds to a local community as we expected.

stable state of a Hopfield net system which is built based on the topology of original network. In the following, we will build the Hopfield net model for the local community extraction problem. As we analyzed in the Supplementary file that the Hopfield net model framework can cover a broad range of quantitative functions, and we will further propose a generic criteria (see *A generic quantitative function* section) to explore the complex hierarchical and multiple-resolution characteristics which can also conquer the resolution-limit of Eq.1 and 2 (see *Resolution limit analysis* section).

Here we briefly describe the classic discrete Hopfield net model [23, 24]. Let's denote a discrete Hopfield net of  $n$  interconnected neurons as  $H = (M, T)$ , where  $M$  is the symmetric weight matrix of size  $n \times n$  and  $T$  is the threshold vector of size  $1 \times n$ . The neuron state vector is denoted as  $x = (x_1, x_2, \dots, x_n)^T \in [0, 1]^n$  and the neurodynamic system can be described by:

$$x(t+1) = \text{sig}\{Mx(t) - T\}, \quad (6)$$

where  $\text{sgn}(x) = (\text{sgn}(x_1), \dots, \text{sgn}(x_n))^T$  and the  $\text{sgn}$  is the signum function defined as  $\text{sgn}(x)$  equal to 1, if  $x \geq 0$ , otherwise 0. The well-known fundamental property of this system is that their dynamics are constrained by an energy function  $E(\mathbf{x})$  (also known as Lyapunov function) defined on their state space by:

$$E(\mathbf{x}) = -\frac{1}{2}\mathbf{x}^T M \mathbf{x} + T^T \mathbf{x}. \quad (7)$$

If the system state converges toward some stable state, then this stable state correspond to a local minimum of  $E(\mathbf{x})$ . It has been proved that the asynchronous Hopfield system starting from any initial state converge to a

stable state provided that  $w(i, i) \geq 0$  for all  $i$ , while the synchronous one converge to a stable state or to a limit cycle of length two under mild hypotheses [25].

We can easily see that the formulaic form of BQP problem is isomorphic to the energy function of Hopfield net, therefore the local community extraction problem defined based on  $Q_S$  and  $W_S$  can be solved by the neurodynamic system. The components of Hopfield nets of them can be defined as  $M_Q = A - \frac{d_i d_j}{2m}$ ,  $T_Q = 0$  and  $M_W = A - \frac{de^T + ed^T}{2n}$ ,  $T_W = -\frac{\lambda}{2}$  respectively (we can omit the scalars). The topology of this system has very natural corresponding relationship with the original network. Particularly, the sparsity of the original network can be employed to accelerate the dynamical update. Moreover, the neurodynamic framework enables us to study the robustness of the local community structure based on the properties of its corresponding stable state.

#### IV. ALGORITHM

The synchronous discrete Hopfield system can be run in a more efficient manner than the asynchronous one, so we adopt it as the basic procedure to solve the BQP problem. The Synchronous Discrete Hopfield Net (named SDHN) can be implemented as follows:

**SDHN** $\{H(M, T)\}$

- **Step 1:** Select an initial vector  $\mathbf{x}(0) \in [0, 1]^n$ , and set  $t = 0$ ;
- **Step 2:** Update  $\mathbf{x}(t)$  by  $\mathbf{x}(t+1) = \text{sig}\{M\mathbf{x}(t) - T\}$ ;

- **Step 3:** If  $\mathbf{x}(t+1)$  satisfies a given stop criterion, then stop and output  $\mathbf{x}(t+1)$ ; else  $t := t+1$  and go to Step 2.

We further employ the SDHN algorithm as subroutine to solve the QFP as follows:

**QFP**( $a(x), b(x)$ )

- **Step 1:** Select an initial vector  $\mathbf{x}(0) \in [0, 1]^n$ , and set  $\lambda(0) = \frac{a(\mathbf{x}(0))}{b(\mathbf{x}(0))}$  and  $k = 0$ .
- **Step 2:** Solve Eq.(5) using  $\text{SDHN}\{H_k(M, T_\lambda)\}$  to get the stable state  $x(k+1)$ .
- **Step 3:** If  $a(\mathbf{x}(k+1)) - \lambda(x_k)b(\mathbf{x}(k+1)) = 0$ , then set  $x^* = x(k+1)$  and  $\lambda^* = \lambda(k)$ , and stop.
- **Step 4:** If  $a(\mathbf{x}(k+1)) - \lambda(x_k)b(\mathbf{x}(k+1)) > 0$ , then set  $\lambda(k+1) = \frac{a(\mathbf{x}(k+1))}{b(\mathbf{x}(k+1))}$ ,  $k = k+1$  and go to Step 2.

Since the topology  $W$  of the Hopfield system only related to the adjacency matrix  $A$  of a network or  $A + B$  where  $B$  is usually of rank 2, then neurodynamic system can be updated in  $O(m+n)$ , where  $m$  is the numbers of edges and  $n$  is the number of nodes in the network. The  $\lambda$  can also be efficiently updated in  $O(m+n)$ . If we take into account the number of iterations  $L$ , we have the computation effort  $O(L(m+n))$  for each run of the neurodynamic system (SDHN procedure). We observed that this procedure usually converges in a small number of iterations (e.g.,  $L = 10$ ). More importantly, we only need to update the  $\lambda$  for **about several times** to converge in running the QFP procedure. Thus the whole neurodynamic framework can be run in a near linear time cost for one trial. Although the neurodynamic system may be trapped in the trivial stable state ( $x = [0, 0, \dots, 0]^T$ ), the high efficiency of this framework makes it possible to run a proper number of trials (e.g., 500). Our neurodynamic procedure is an very efficient algorithm which enable it to be a powerful way to extract the local communities in a large-scale network. While the stochastic search methods like tabu search approach are highly time-consuming techniques which can only be applied to network of at most a few thousand vertices with common hardware.

After determining a local community, we can further apply the whole framework to its complement in the network to extract next community. A challenging and open problem faced by all methods for community-detection is how to determine the number of communities in a network. In real applications, we would suggest to evaluate the statistical significance of a community by comparing its objective value with that of 100 random networks generated by containing the same set of nodes and the same number of edges [26].

## V. RESOLUTION LIMIT ANALYSIS

Unfortunately, due to the improper penalty concerning to the total size of networks, this local modularity crite-

on has serious resolution limit as found for the popular modularity function [4, 8]. This resolution limit problem of modularity has been carefully discussed by Fortunato and Barthelemy [9]. To illustrate the resolution limit of the local modularity function, we analyze the local modularity of ‘communities’ in several schematic examples. The ring-clique network consisting of a ring of cliques and connecting through single links adopted by Fortunato and Barthelemy in ref. [9] is employed (Figure 2A); each clique is a complete graph  $K_m$  with  $m$  nodes and has  $m(m-1)/2$  links. If we assume that there are  $n$  cliques (with  $n$  even), the network has a total of  $N = nm$  nodes and  $L = nm(m-1)/2 + n$  links. Obviously, this network has a clear community structure where the communities correspond to single cliques, and we expect that any community extraction algorithm should be able to detect these communities.

The local modularity  $W_{\text{clique}}$  of a clique can be easily calculated and is equal to

$$W_{\text{clique}} = m(m-1)(n-1) - 2.$$

On the other hand, the local modularity  $W_{\text{pairs}}$  of the pairwise consecutive cliques are considered as single communities (as shown by the dotted lines in Figure 2A) is

$$W_{\text{pairs}} = m(m-1)(n-2) + n - 4.$$

The difference  $\Delta W = W_{\text{clique}} - W_{\text{pairs}}$  is

$$\Delta W = m(m-1) + 2 - n,$$

Then the condition  $W_{\text{clique}} > W_{\text{pairs}}$  is satisfied only if

$$m(m-1) + 2 > n, \quad (8)$$

Obviously,  $m$  and  $n$  are independent variables, and we can choose them such that the inequality of Eq.8 is not satisfied. For instance, for  $m = 10$  and  $n = 100$ ,  $W_{\text{clique}} = 8908$  and  $W_{\text{pairs}} = 8916$ . Then an efficient algorithm would find the configuration with pairs of cliques and not the single clique as an ‘ideal’ community to get the maximum local modularity. As we can see that, as  $n$  increases, the difference  $-\Delta W$  becomes even larger.

This situation can also be observed in another example (shown in Figure 2B), where the two pink circles again represent cliques with  $p$  nodes linked each other by a single edge, the other part is the rest of the network with  $n$  nodes linked these two cliques by single link respectively. We can similarly calculate the local modularity of a clique as  $W_{\text{clique}} = (n+p)(p-1) - 2$  and the pair cliques as  $W_{\text{pairs}} = n(p-1) + n/p - 2$ . Then the difference  $\Delta W = W_{\text{clique}} - W_{\text{pairs}}$  is  $\Delta W = \frac{p^2(p-1)-n}{p}$ . So if  $p^2(p-1) < n$ , the two cliques will merge as one community. For example, if we take the network of Figure 2B with  $p = 10$  and  $n = 1000$  and the  $n$  nodes was randomly connected with probability  $p = 0.05$ , we have seen that the local modularity criterion will get a ‘module’ which are pairs of connected cliques. Although the example

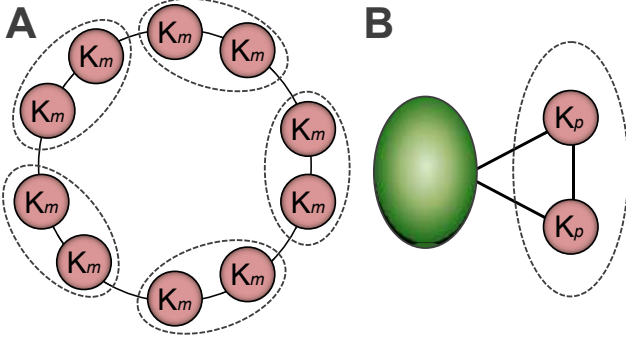


FIG. 2: Illustrative examples. (A) A network composed of identical cliques with  $m$  nodes connected by single links. If the number of cliques  $n$  is larger than about  $m(m-1)+2$ , the local modularity optimization would lead to a local community which are combined of two or more cliques (represented by dotted lines). (B) A network with two identical cliques of  $p$  nodes linked by a single link between them, and other subnetwork of  $n$  nodes with arbitrary connections; the two cliques are linked to the subnetwork with a single link respectively. If  $n$  is large enough with respect to  $p$  (e.g.,  $n = 1000$ ,  $p = 10$ ), the local modularity optimization merges the two cliques into one (shown with a dotted line).

was very simple, we can clearly see that the local community was only affected by the number of nodes in the rest of the network, while not the specific structure of it. This means that, in large-scale sparse networks, the local dense communities are easily to be merged together which was also further observed in our simulation study as well as in real networks.

## VI. A GENERIC QUANTITATIVE FUNCTION

As discussed by Zhao et al. [13], the factor  $|S||S^\rho|$  in  $W_S$  tend to extract larger communities and avoid the smaller communities without this factor. However, this factor relate a local community with the size of network (similar to  $Q$  related with number of total links) which further lead to the resolution limit. Taking into account the resolution limit of  $W_S$ , we propose a parametric quantitative criterion to extract local communities and explore multi-resolution community characteristic of networks. The generic quantitative function  $W_S^\rho$  is defined as follows by introducing a parameter  $\rho$ :

$$W_S^\rho = |S||S^\rho| \left[ \frac{O_S}{|S|^2} - \frac{B_S}{|S||S^\rho|} \right], \quad (9)$$

where  $|S^\rho| = \rho n - |S|$ ,  $2\frac{|S|}{n} < \rho \leq 1$ . The  $|S^\rho|$  can be considered as the estimation of the number of nodes connecting to the community  $S$  in the rest of the network.  $[2\frac{|S|}{n} < \rho \leq 1]$ , and  $\langle S \rangle$  is the expected number of community size. Then if the factor  $|S| = \frac{\rho n}{2}$ ,  $|S||S^\rho|$  gets its maximum, i.e.,  $\rho = \frac{2|S|}{n}$ . If  $\rho = \frac{2|S|}{n}$ , the criterion is

become  $O_S - B_S$  which is related to the weak definition [16], while if  $\rho = 1$ , the criterion is just the  $W_S$ . Similar with  $W_S$ , the generic function  $W_S^\rho$  can be solved by the topology-varying neurodynamic model efficiently. In particular, the components of corresponding Hopfield net can be defined as  $M_{W^\rho} = \rho A - \frac{de^T + ed^T}{2n}$ ,  $T_W = -\frac{\lambda}{2}$  (we can omit the scalars). In real applications, we can sample  $\rho$  from the given range to study the detailed community structure of networks systematically.

## VII. SIMULATION

We generate a network consisting two tight communities and weakly connected background as suggested by Zhao et al. [13]. We consider community sizes  $n_1 = 100$  and  $n_2 = 200$ , which are embedded into the background nodes forming the network of size  $n$  ( $n = 1000$  and  $10000$ ). The links between a community, and the links between members and others or links between nodes in the background all form independently with probability 0.3 and 0.05. We compare the modularity partition (with spectral optimization [8]) and the local community extraction with  $W_S(\rho = 1)$  and  $W_S^\rho = 0.6$ . We partition the network into three parts by modularity and extract two communities by the extraction methods. We show the accuracy to compare these networks.

We should note that the parameter is tunable and we can obtain a spectrum of ‘local’ communities by sampling  $\rho$  in the range  $[\rho_{min}, 1]$ . Such spectrum can shed light on the underlying multiple resolution or hierarchical community structure of networks. To show this point, we simulated a network of size  $n = 1000$  with multiple resolution community structure. The spectrum constructed by our method with different  $\rho$  can well uncover such complicated communities in the network. We believe this will benefit the understanding of the underlying mesostructure of networks.

We have shown that the neurodynamic framework can accurately identify the communities in the simulated networks with significantly higher precision than that of the original one and the modularity, and the generic quantitative criterion can conquer the resolution limit of the original one. As we have analyzed that the cost of the neurodynamic procedure is only about  $O(T(m+n))$ . Here the  $T$  is about 10-20 for network with  $n$  between 100 to 10000. We further estimate how many trials must be used to identify the optimal community. We found 500 random trials are enough to get the best solution. We have compare the time cost of our procedure with the tabu search for maximize  $W_S$ . The tabu search can only be applied to the network with thousands of nodes, while our procedure can work on the network with 20000 nodes in 30 seconds. The efficiency of our method make it feasible to real large-scale networks.

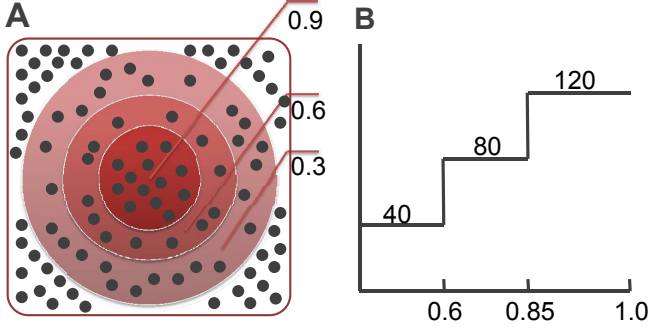


FIG. 3: (A) The local communities identified by our approach in the simulated network with multiple-resolution community structure. Different colors illustrate the membership of each local community. (B) The membership of local communities identified by our approach with different  $\rho$ .

## VIII. EXAMPLE APPLICATIONS

**Two well-known examples with significant community structure.** The first example is the well-known network of friendship between 34 members of a karate club at an American university [27]. This network is of particular interest because the club split into two sub-clubs due to an internal dispute, and the structure of the recorded network reflects the trend of this division. The second example is the network representing the schedule of Division I football games for the 2000 season [4]. The network consisting of 115 nodes and 613 links representing teams and regular season games between two teams, are divided into conferences with more games are played within conferences than across them.

The partitioning methods including modularity can partition these two networks into (almost) the exact known groups [8]. However, the local community extraction method reveals the ‘core’ structural information about these (this) networks. Figure 4A shows the karate club network and its local communities extracted by our method with  $\rho = 1$ , which are consistent with the results obtained by optimizing the criterion using taboo search technique [13]. We can observe that the first two communities occupy the core part of the original two groups which determine the evolving trend, and the third one is a subcommunity in the **instructor group** which is prone to a further division. We should note that although the order of objective values have changed, we can get the same local communities with  $\rho$  ranging from 0.6 to 1 (Figure 4B).

However, when we apply the  $W_S$  (i.e.,  $W_S^\rho$  with  $\rho = 1$ ) to the football network, we can observe clear resolution limit problem. Figure 4 show the extracted communities with  $\rho = 1$  and  $\rho = 0.4$ . The local communities identified with  $W_S$  tend to combine several groups together which clearly show the resolution limit of the quantitative function  $W_S$ . While our neurodynamic system based on the

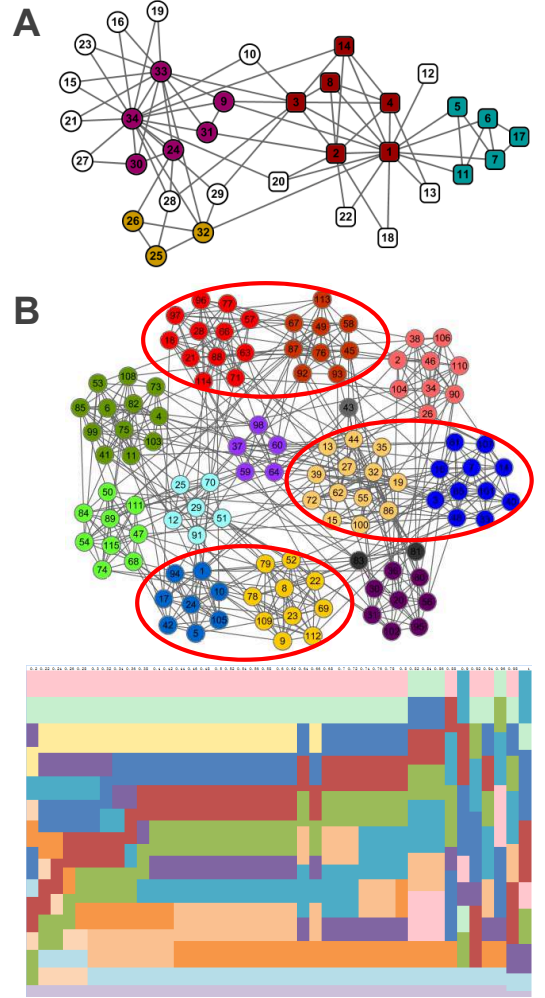


FIG. 4: (A) The local communities identified by our approach in the karate club network. Shapes of nodes indicate the membership of the corresponding individuals in the two known factions of the network. Different colors illustrate the membership of each local community. (B) The local communities identified by our approach in the football network with  $\rho = 0.4$  and  $\rho = 1$  and the membership of local communities identified by our approach with different  $\rho$ .

generic function with  $\rho = 0.4$  can well identify those local communities that correspond to the **core part of** real groups well.

## IX. CONCLUSION

We have shown that the process of resolving local communities in complex networks can be viewed as finding stable states in a neurodynamic system. By describing local communities with a quantitative function, we are able to construct a corresponding neurosystem which can store the structural patterns as stable states among it.



The solution of optimizing the quantitative function for describing local communities is, in principle, NP-hard. However, the neurodynamic system can be run in a very efficient manner which makes it applicable to large-scale networks, while the heuristic search method can not (e.g., tabu search used by ref. [13]). The original local community criterion proposed by Zhao *et al.* [13] was also hindered by serious resolution limit issue as showed for modularity [9]. Actually, as we stated that any quantitative function related to the whole size of network (the total number of nodes or links) will bear this problem. A generic parametric quantitative function is in demand which can help to explore the complicated multi-resolution structure of networks. We believe that the local community extraction framework proposed here is an important complementary to the traditional partitioning methods which may bear distinct underlying

problem due to the global modular hypothesis. We expect the proposed method will benefit network science with broad applications in various fields including biology, sociology and technology.

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## *Abstract*



